Introduction

2D graphics models may combine geometric models (also called vector graphics), digital images (also called raster graphics), text to be typeset (defined by content, font style and size, color, position, and orientation), mathematical functions and equations, and more. These components can be modified and manipulated by two-dimensional geometric transformations such as translation, rotation, scaling.

Matrices

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$$

- A matrix is a rectangular array of numbers.
- A general matrix will be represented by an upper-case italicised letter.
- The element on the *i*th row and *j*th column is denoted by $a_{i,j}$. Note that we start indexing at 1, whereas *C* indexes arrays from 0.

Matrices – Addition

- Given two matrices *A* and *B* if we want to add *B* to *A* (that is form *A*+*B*) then if *A* is ($n \times m$), *B* must be ($n \times m$), Otherwise, *A*+*B* is not defined.

- The addition produces a result, *C* = *A*+*B*, with elements:

$$C_{i,j} = A_{i,j} + B_{i,j}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

Matrices – Multiplication

- Given two matrices *A* and *B* if we want to multiply *B* by *A* (that is form *AB*) then if *A* is ($n \times m$), *B* must be ($m \times p$), i.e., the number of columns in *A* must be equal to the number of rows in *B*. Otherwise, *AB* is not defined.

- The multiplication produces a result, $C = AB$, with elements:

$$C_{i,j} = \sum_{k=1}^{m} a_{ik} b_{kj}$$

(Basically we multiply the first row of *A* with the first column of *B* and put this in the $c_{1,1}$ element of *C*. And so on…).

Matrices – Multiplication (Examples)

$\boxed{2\times6+6\times3+7\times2=44}$

$$\begin{bmatrix} 2 & 6 & 7 \\ 4 & 5 & 8 \\ 9 & 2 & 3 \end{bmatrix} \times \begin{bmatrix} 6 & 8 \\ 3 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 44 & 76 \\ 55 & 95 \\ 66 & 96 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 \\ 4 & 5 \end{bmatrix} \times \begin{bmatrix} 6 & 8 \\ 3 & 3 \\ 2 & 6 \end{bmatrix}$$

Undefined!

2x2 x 3x2 2!=3

2x2 x 2x4 x 4x4 is allowed. Result is 2x4 matrix

Matrices -- Basics

- Unlike scalar multiplication, $AB \neq BA$

- Matrix multiplication distributes over addition:

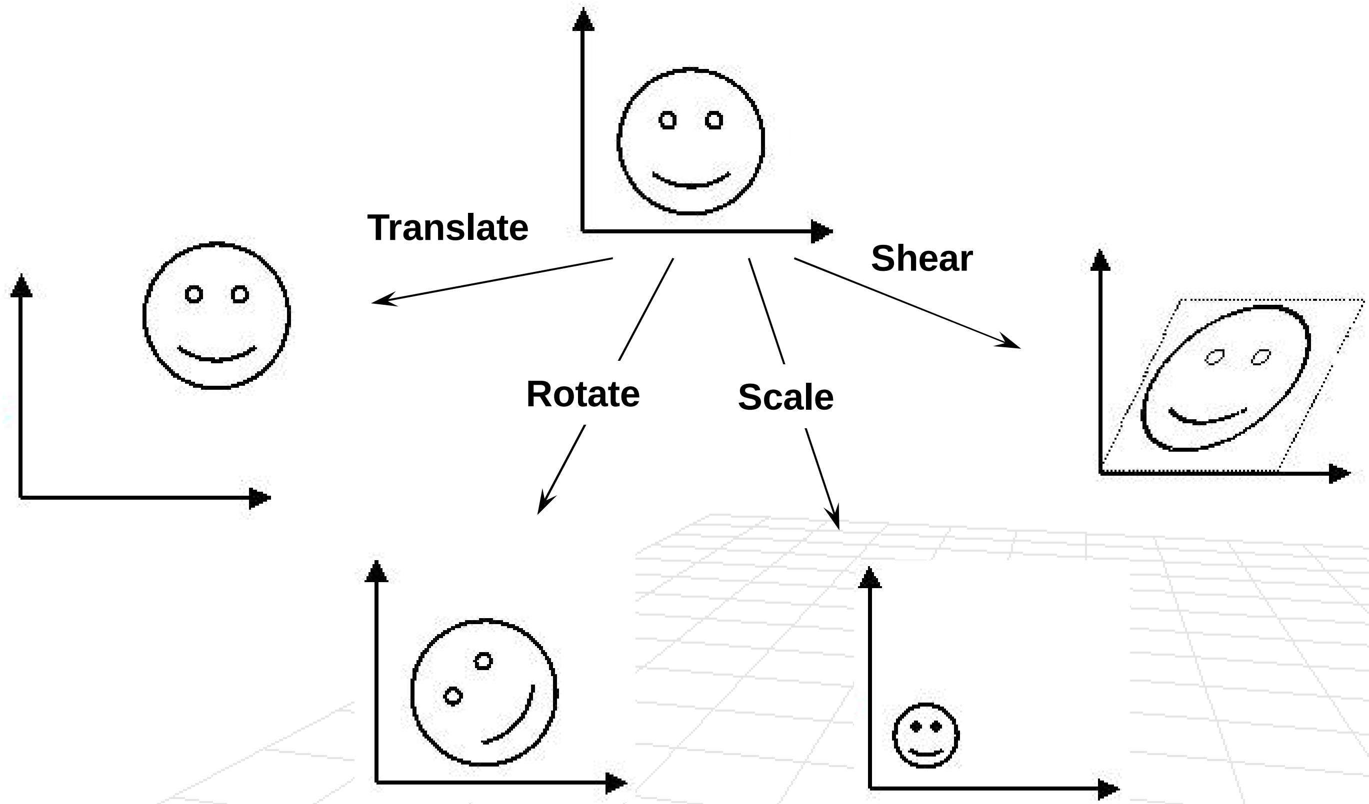
$$A(B+C) = AB + AC$$

- Identity matrix for multiplication is defined as I.

- The transpose of a matrix, A, is either denoted A^T or A' is obtained by swapping the rows and columns of A:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix} \Rightarrow A' = \begin{bmatrix} a_{1,1} & a_{2,1} \\ a_{1,2} & a_{2,2} \\ a_{1,3} & a_{2,3} \end{bmatrix}$$

2D Geometrical Transformations

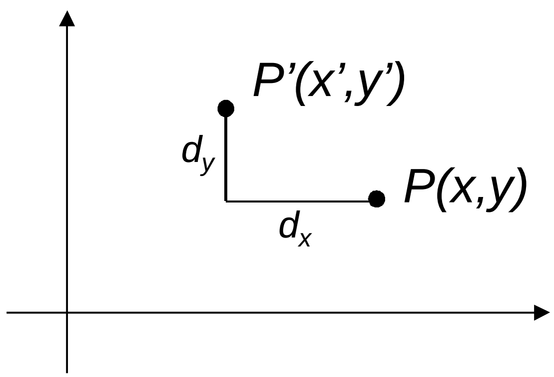


Translate Points

Recall.. We can translate points in the (*x, y*) plane to new positions by adding translation amounts to the coordinates of the points. For each point *P*(*x, y*) to be moved by d_x units parallel to the *x* axis and by d_y units parallel to the *y* axis, to the new point *P'*(*x', y'*). The translation has the following form:

$$x' = x + d_x$$

$$y' = y + d_y$$



In matrix format:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

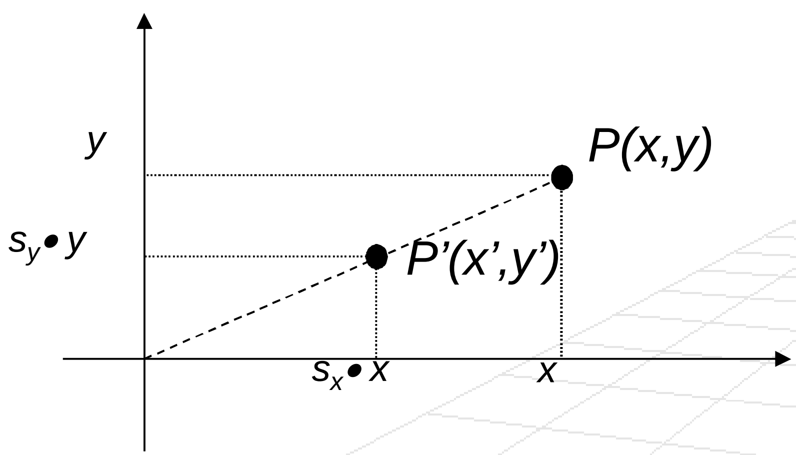
If we define the translation matrix $T = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$, then we have *P' =P + T.*

Scale Points

Points can be scaled (stretched) by s_x along the x axis and by s_y along the y axis into the new points by the multiplications:

We can specify how much bigger or smaller by means of a "scale factor"

To double the size of an object we use a scale factor of 2, to half the size of an obejct we use a scale factor of 0.5



$$x' = s_x x$$

$$y' = s_y y$$

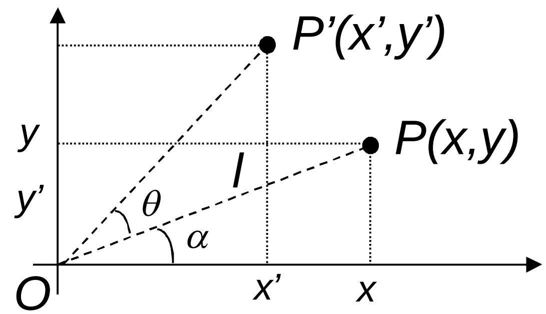
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

If we define $S = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$, then we have $P' = SP$

Rotate Points (*cont.*)

Points can be rotated through an angle θ about the origin:

$$|OP'| = |OP| = l$$



$$x' = |OP'|\cos(\alpha+\theta) = l\cos(\alpha+\theta)$$
$$= l\cos\alpha\cos\theta - l\sin\alpha\sin\theta$$
$$= x\cos\theta - y\sin\theta$$

$$y' = |OP'|\sin(\alpha+\theta) = l\sin(\alpha+\theta)$$
$$= l\cos\alpha\sin\theta + l\sin\alpha\cos\theta$$
$$= x\sin\theta + y\cos\theta$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{P' = RP}$$

Review…

- Translate: $P' = P+T$
- Scale: $P' = SP$
- Rotate: $P' = RP$

Application

Many graphical user interfaces (GUIs), including Mac OS, Microsoft Windows, or the X Window System, are primarily based on 2D graphical concepts. Such software provides a visual environment for interacting with the computer, and commonly includes some form of window manager to aid the user in conceptually distinguishing between different applications. The user interface within individual software applications is typically 2D in nature as well, due in part to the fact that most common input devices, such as the mouse, are constrained to two dimensions of movement.

Scope of Research

A large market for 3D models (as well as 3D-related content, such as textures, scripts, etc.) still exists - either for individual models or large collections. Online marketplaces for 3D content, such as Turbo Squid and DAZ 3D, allow individual artists to sell content that they have created. 3D printing is a form of additive manufacturing technology where a three dimensional object is created by laying down successive layers of material.